PART 1: QUESTIONS

Name:______________________________ **Age:**_____ **Id:**_______________ **Course:**__________

Derivatives - Exam 1 **Lesson: 1-3**

Instructions:

- Please begin by printing your Name, your Age, your Student Id , and your Course Name in the box above and in the box on the solution sheet.
- You have 90 minutes (class period) for this exam.
- You can not use any calculator, computer, cellphone, or other assistance device on this exam. However, you can set our flag to ask permission to consult your own one two-sided-sheet notes at any point during the exam (You can write concepts, formulas, properties, and procedures, but questions and their solutions from books or previous exams are not allowed in your notes).
- Each multiple-choice question is worth 5 points and each extra essay-question is worth from 0 to 5 points. (Even a simple related formula can worth some points).
- Set up your flag if you have a question.
- Relax and use strategies to improve your performance.

Exam Strategies to get the best performance:

- Spend 5 minutes reading your exam. Use this time to classify each Question in (E) Easy, (M) Medium, and (D) Difficult.
- Be confident by solving the easy questions first then the medium questions.
- Be sure to check each solution. In average, you only need 30 seconds to test it. (Use good sense).
- Don't waste too much time on a question even if you know how to solve it. Instead, skip the question and put a circle around the problem number to work on it later. In average, the easy and medium questions take up half of the exam time.
- Solving the all of the easy and medium question will already guarantee a minimum grade. Now, you are much more confident and motivated to solve the difficult or skipped questions.
- Be patient and try not to leave the exam early. Use the remaining time to double check your solutions.

1. The derivative of a function is:

I. another function. For example, the derivative of the position function of an object is the velocity function that measures how quickly the position changes.

II. the slope of the tangent line to the graph of the function at that point.

III. defined as the limit $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$.

IV. the instantaneous rate of change of the function.

Then:

a) All alternatives are correct.
b) All alternatives are **incorrect**.
c) Only I and II are correct.
d) Only I and III are correct.
e) Only II and III are correct.

Solution: a

All alternatives are correct.

2. The derivative of $f(x)$ with respect to x is the function $f'(x)$ and is defined as,

a) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

b) $f'(x) = \lim_{h \to \infty} \dfrac{f(x+h) - f(x)}{h}$

c) $f'(x) = \lim_{h \to 0} \dfrac{f(x) - f(x+h)}{h}$

d) $f'(x) = \lim_{h \to \infty} \dfrac{f(x) - f(x+h)}{h}$

e) All of the above.

Solution: a

The derivative of $f(x)$ with respect to x is the function $f'(x)$ and is defined as,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

3. Let $f(x)$ be a differential function on x. The notation for the first, second and third derivatives are respectively:

I. $f'''(x), f''(x),$ and $f'(x)$.

II. $\dfrac{d^3f}{dx^3}, \dfrac{d^2f}{dx^2},$ and $\dfrac{df}{dx}$.

III. $f'(x), f''(x),$ and $f'''(x)$.

IV. $\dfrac{df}{dx}, \dfrac{d^2f}{dx^2},$ and $\dfrac{d^3f}{dx^3}$.

a) Only I and II are correct.
b) Only III and IV are correct.
c) Only I and IV are correct.
d) Only II and III are correct.
e) None of the above.

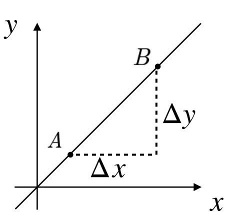
Solution: c

The notation for the first, second and third derivatives are respectively:

$$\frac{df}{dx} \text{ or } f'(x), \ \frac{d^2f}{dx^2} \text{ or } f''(x), \text{ and } \frac{d^3f}{dx^3} \text{ or } f'''(x).$$

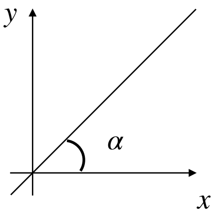
4. The slope of a straight line is:

I.



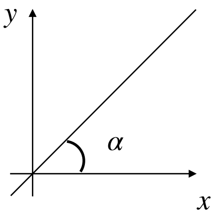
$$m = \frac{\Delta y}{\Delta x} = \frac{y_B - y_A}{x_B - x_A}$$

II.



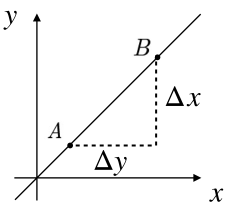
$\mathrm{m} = \tan \alpha$

III.



$\mathrm{m} = \cot \alpha$

IV.

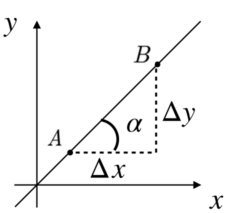


$\mathrm{m} = \frac{\Delta \mathrm{x}}{\Delta \mathrm{y}} = \frac{\mathrm{x_B - x_A}}{\mathrm{y_B - y_A}}$

a) Only I and III are correct.
b) Only I and II are correct.
c) Only II and III are correct.
d) Only II and IV are correct.
e) Only III and IV are correct.

Solution: b

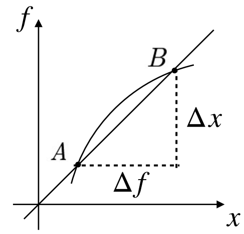
The slope of a straight line is:



$\mathrm{m} = \frac{\Delta \mathrm{y}}{\Delta \mathrm{x}} = \frac{\mathrm{y_B - y_A}}{\mathrm{x_B - x_A}} = \tan \alpha$

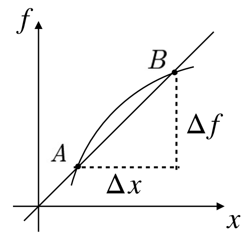
5. The derivative of a curve $f(x)$ on x is:

a)



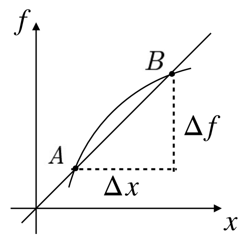
$\frac{dx}{df} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta f} = \lim_{\Delta x \to 0} \frac{\Delta x}{f(x+\Delta x) - f(x)}$

b)



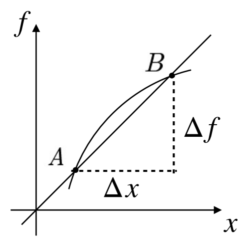
$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

c)



$\frac{df}{dx} = \lim_{\Delta x \to \infty} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to \infty} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

d)

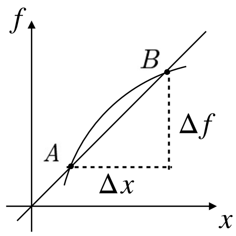


$\frac{df}{dx} = \frac{\Delta f}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$

e) None of the above.

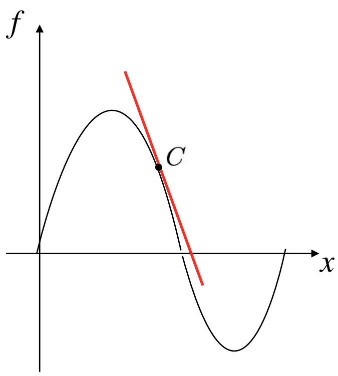
Solution: b

The derivative of a curve $f(x)$ on x is:



$$\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

6. Given the graph of f on x.



a) $f'(x_C) = 0 \Rightarrow f$ has a local maximum at C.

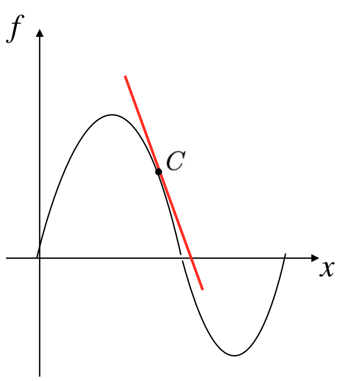
b) $f'(x_C) = 0 \Rightarrow f$ has a local minimum at C.

c) $f'(x_C) < 0 \Rightarrow f$ is decreasing at C.

d) $f'(x_C) > 0 \Rightarrow f$ is increasing at C.

e) None of the above.

Solution: c



$f'(x_C) < 0 \Rightarrow f$ is decreasing at C.

7. The velocity of a car is given by the following function:

$$v(t) = t^3 - t^2 + 4t \ \ (m/s)$$

The car acceleration at $t = 2\ s$ is:

a) $9\ m/s^2$.
b) $10\ m/s^2$.
c) $11\ m/s^2$.
d) $12\ m/s^2$.
e) None of the above.

Solution: d

$v(t) = t^3 - t^2 + 4t \ \ (m/s)$

$a(t) = v'(t) = 3t^2 - 2t + 4$

$a(2) = 3(2)^2 - 2(2) + 4 = 12\ m/s^2$.

8. Given $f(x) = 2x^4 + x^3 - 3x^2 + x + \pi$. Then the derivative $f'(x)$ is:

a) $8x^3 + 3x^2 - 6x + \pi$

b) $8x^3 + 3x^2 - 6x - \pi$

c) $8x^3 + 3x^2 + 6x + \pi$

d) $8x^3 - 3x^2 - 6x + \pi$

e)None of the above.

Solution: e

Let $f(x) = 2x^4 + x^3 - 3x^2 + x + \pi$.

Then the derivative $f'(x)$ is:

$$f'(x) = (4)(2)x^{4-1} + 3(1)x^{3-1} - (2)(3)x^{2-1} + (1)(1)x^{1-1}$$
$$= 8x^3 + 3x^2 - 6x + 1.$$

9. Given $f(x) = (x^2 + 3x)(x - 2x^2)$.

The derivative $f'(x)$ using product rule is:

a) $f'(x) = (2x + 3)(x + 2x^2) + (x^2 + 3x)(1 - 4x)$.
b) $f'(x) = (2x + 3)(x - 2x^2) + (x^2 + 3x)(1 + 4x)$
c) $f'(x) = (2x + 3)(x - 2x^2) - (x^2 + 3x)(1 + 4x)$
d) $f'(x) = (2x - 3)(x - 2x^2) + (x^2 + 3x)(1 - 4x)$
e) None of the above.

Solution: c

Product rule: $f(x) = uv \Rightarrow f'(x) = u'v + uv'$.

Let $f(x) = (x^2 + 2x)(x - 2x^2)$.
$f'(x) = (2x - 3)(x + 2x^2) - (x^2 - 3x)(1 + 4x)$.

10. Given $f(x) = \dfrac{(x^2 + 3x)}{(x + 2x^2)}$.

The derivative $f'(x)$ using quotient rule is:

a) $f'(x) = \dfrac{(2x + 3)(x + 2x^2) - (x^2 + 3x)(1 + 4x)}{(x + 2x^2)^2}$

b) $f'(x) = \dfrac{(2x + 3)(x + 2x^2) + (x^2 + 3x)(1 + 4x)}{(x + 2x^2)^2}$

c) $f'(x) = \dfrac{(2x + 3)(x + 2x^2) - (x^2 + 3x)(1 - 4x)}{(x + 2x^2)^2}$

d) $f'(x) = \dfrac{(2x + 3)(x + 2x^2) + (x^2 + 3x)(1 - 4x)}{(x + 2x^2)^2}$

e) None of the above.

Solution: a

Quotient rule: $f(x) = \dfrac{u}{v} \Rightarrow f'(x) = \dfrac{u'v - uv'}{v^2}$.

Let $f(x) = \dfrac{(x^2 + 3x)}{(x + 2x^2)}$.

$f'(x) = \dfrac{(2x + 3)(x + 2x^2) - (x^2 + 3x)(1 + 4x)}{(x + 2x^2)^2}$.

11. Given $f(x) = \tan x$.

The derivative $f'(x)$ is:

a) $f'(x) = \cot x$
b) $f'(x) = \sec x$
c) $f'(x) = -\sin x$
d) $f'(x) = -\sec x$
e) None of the above.

Solution: e

The derivative of $f(x) = \tan x$ is $f'(x) = \sec^2 x$.

12. Given $f(x) = \log_3 x$.

The derivative $f'(x)$ is:

a) $f'(x) = \dfrac{1}{x} \log_e 3$

b) $f'(x) = \dfrac{1}{x} \log_3 e$

c) $f'(x) = \dfrac{1}{x} \log 3$

d) $f'(x) = \log_3 x$

e) None of the above.

Solution: b

$y = \log_a x \Rightarrow y' = \dfrac{1}{x} \log_a e$.

Let $f(x) = \log_3 x$.

The derivative of $f(x)$ is $f'(x) = \dfrac{1}{x} \log_3 e$.

13. Given $f(x) = \pi^x$. The derivative $f'(x)$ is:

a) $f'(x) = e^x \log_e \pi$
b) $f'(x) = \pi^x \log_e \pi$
c) $f'(x) = \pi^x \log_\pi e$
d) $f'(x) = \pi^x$
e) None of the above.

Solution: c

$y = a^x \Rightarrow y' = a^x \log_e a.$

Let $f(x) = \pi^x$.

The derivative of $f(x)$ is $f'(x) = \pi^x \log_e \pi$.

14. Given $f(x) = \sqrt[4]{x}$.

The derivative $f'(x)$ is:

a) $f'(x) = \dfrac{1}{4\sqrt[4]{x}}$

b) $f'(x) = \dfrac{1}{4\sqrt[4]{x^3}}$

c) $f'(x) = \dfrac{1}{\sqrt[4]{x^3}}$

d) $f'(x) = \dfrac{1}{4\sqrt[2]{x^4}}$

e) None of the above.

Solution: b

$y = x^n \Rightarrow y' = n x^{n-1}$.

Let $f(x) = \sqrt[4]{x} = (x)^{\frac{1}{4}}$.

The derivative of $f(x)$ is:

$$f'(x) = \frac{1}{4}(x)^{\frac{1}{4}-1} = \frac{1}{4}(x)^{-\frac{3}{4}} = \frac{1}{4(x)^{\frac{3}{4}}} = \frac{1}{4\sqrt[4]{x^3}}.$$

15. Given:

I. The Chain Rule is used to find the derivative of a composite function.

II. The Chain Rule can be expressed as:

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x).$$

III. The Chain Rule (Fast Method) derives a complex function from the outer function to the inner one $\mathrm{v(u)' = v'(u)u'}$.

a) I, II, and III are **incorrect**.
b) I, II, and III are correct.
c) Only I and II are correct.
d) Only I and III are correct.
e) Only II and III are correct.

Solution: b

All alternatives are correct.

16. The derivative of $f(x) = \sin[\sin(x)]$ is:

a) $f'(x) = \cos[\cos(x)]\sin(x)$

b) $f'(x) = \cos[\sin(x)]\sin(x)$

c) $f'(x) = -\cos[\sin(x)]\cos(x)$

d) $f'(x) = -\cos[\cos(x)]\sin(x)$

e) None of the above.

Solution: e

Let $f(x) = \sin(x)$ and $g(x) = \sin x$.

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x).$$

$f'(x) = \cos[\sin(x)]\cos(x)$.

17. The derivative of $f(x) = \log_e[\tan x]$ is:
Assume $\tan x > 0$.

a) $f'(x) = \cot x$

b) $f'(x) = 3\cot(3x)$

c) $f'(x) = -\tan x$

d) $f'(x) = -3\tan(3x)$

e) None of the above.

Solution: e

Existence condition: $\tan x > 0$.

Let $f(x) = \log_e[\tan x]$. By the Chain Rule, we have:

$$f'(x) = \frac{1}{\tan x}(\log_e e)\ \sec^2 x = \frac{\sec^2 x}{\tan x}.$$

18. The derivative of $f(x) = (x^2 - 4x + \pi)^7$ is:

a) $f'(x) = 7(x^2 - 4x + \pi)^6$

b) $f'(x) = (x^2 - 4x + \pi)^6(2x - 4)$

c) $f'(x) = 7(x^2 - 4x + \pi)(2x - 4)$

d) $f'(x) = 7(x^2 - 4x + \pi)^6(2x - 4)$

e) None of the above.

Solution: d

Let $f(x) = (x^2 - 4x + \pi)^7$.

By the Chain Rule, we have:

$$f'(x) = 7(x^2 - 4x + \pi)^6(2x - 4).$$

19. The derivative of $f(x) = 4^{(x^2 - 2x + e)}$ is:

a) $f'(x) = 4^{(x^2-2x+e)}(\log_e 4)$

b) $f'(x) = 4^{(x^2-2x+e)}(2x - 2)$

c) $f'(x) = 4^{(x^2-2x+e)}(\log_e 4)(2x - 2)$

d) $f'(x) = 4^{(x^2-2x+e)}(\log_4 e)(2x - 2)$

e) None of the above.

Solution: c

Let $f(x) = 4^{(x^2-2x+e)}$. By the Chain Rule, we have:

$$f'(x) = 4^{(x^2-2x+e)}(\log_e 4)(2x - 2).$$

20. The derivative of $f(x) = 7^{\tan(x^2)}$ is:

a) $f'(x) = 7^{\tan(x^2)}(\log_e 7)\sec^2(x^2)$.

b) $f'(x) = 7^{\tan(x^2)}(\log_e 7)\sec^2(x^2)2x$.

c) $f'(x) = 7^{\tan(x^2)}(\log_7 e)\sec(x^2)2x$.

d) $f'(x) = 7^{\tan(x^2)}\sec^2(x^2)2x$.

e) None of the above.

Solution: b

Let $f(x) = 7^{\tan(x^2)}$.

By the Chain Rule, we have:

$$f'(x) = 7^{\tan(x^2)}(\log_e 7)\sec^2(x^2)2x.$$

PART 2: SOLUTIONS

Consulting

Name:________________________ **Age:**_____ **Id:**________________ **Course:**__________

Multiple-Choice Answers

Questions	A	B	C	D	E
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
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	Points	Max
Multiple Choice		100
Extra Points		25
Consulting		10
Age Points		25
Total Performance		160
Grade		A

Extra Questions

21. Let $A = L^2 + L$ be the area of a rectangle of sides L and $L+1$. The measure L changes over time according to the function $L(t) = t^2$ feet (t in minutes).
Find $\frac{dA}{dt}$ when $t = 1$ min.

Solution: $\frac{dA}{dt} = 6\frac{ft^2}{min}$

$$A = L^2 + L \Rightarrow \frac{dV}{dL} = 2L + 1 = 2t^2 + 1.$$

$L(t) = t^2 \Rightarrow \frac{dL}{dt} = 2t$. Then

$$\frac{dA}{dt} = \frac{dA}{dL}\frac{dL}{dt} = (2t^2+1)(2t) = 4t^3 + 2t$$

For $t = 1$s, we have:

$$\frac{dA}{dt} = 4(1)^3 + 2(1) = 6\frac{ft^2}{min}.$$

22. Find the derivative of $f(x) = \log_2\left[10^{\cos(x)}\right]$.

Solution:

$$f'(x) = \left[\frac{1}{10^{\cos(x)}}\log_2 e\right]\left[10^{\cos(x)}\log_e 10\right]\left[-\sin(x)\right].$$

$$f(x) = \log_2\left[10^{\cos(x)}\right]$$

$$f'(x) = \left[\frac{1}{10^{\cos(x)}}\log_2 e\right]\left[10^{\cos(x)}\log_e 10\right]\left[-\sin(x)\right].$$

23. Given $f(x) = x^2$.

Find the derivative of $f(x)$ by the definition.

Hint: The definition of the derivative of $f(x)$ is:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Solution: $f'(x) = 2x$.

Let $f(x) = x^2$. Then:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow$$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \Rightarrow$$

$$f'(x) = \lim_{h \to 0} \frac{h(2x+h)}{h}$$

$$f'(x) = \lim_{h \to 0} (2x+h) \Rightarrow f'(x) = 2x.$$

24. A car moves in a road according of the position function $s(t) = t^4 - t^3$ (s in meters and t in seconds).

Find the acceleration when $t = 1s$.

Hint: Acceleration $a(t) = s''(t)$.

Solution: $a(1) = 6\ m/s^2$.

Let $s(t) = t^4 - t^3 \Rightarrow s'(t) = 4t^3 - 3t^2 \Rightarrow$

$s''(t) = 12t^2 - 6t \Rightarrow a(t) = 12t^2 - 6t$.

Then $a(t) = 12(1)^2 - 6(1) \Rightarrow a(1) = 6\ m/s^2$.

25. Given $f(x) = \sqrt[7]{x}$.

Find the derivative of $f(x)$.

Solution: $f'(x) = \dfrac{1}{7\sqrt[7]{x^6}}$.

let $f(x) = \sqrt[7]{x}$. Then $f(x) = x^{\frac{1}{7}}$.

$$f'(x) = \frac{1}{7}x^{\left(\frac{1}{7}-1\right)} = \frac{1}{7}x^{\left(-\frac{6}{7}\right)} = \frac{1}{7x^{\frac{6}{7}}} = \frac{1}{7\sqrt[7]{x^6}}.$$